Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

AS
MATHEMATICS
Unit Further Pure 1

Wednesday 13 June 2018 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Answer **all** questions.

Answer each question in the space provided for that question.

1 A curve C_1 has equation $xy = 16$.

(a) Sketch the curve C_1 .

[1 mark]

(b) The curve C_1 is translated by the vector $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ to give the curve C_2 .

(i) Find an equation of C_2 .

[1 mark]

(ii) Write down the equations of the asymptotes of C_2 .

[1 mark]

(c) The curve C_2 is reflected in the line $y = x$ to give the curve C_3 . Find an equation of C_3 .

[1 mark]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

2 The equation

$$x - x^2 + \frac{2}{x} + \frac{3}{2} = 0$$

has one real positive root, α.

(a) Show that α lies in the interval $2 < \alpha < 2.5$.

[2 marks]

(b) Taking $x_1 = 2$ as a first approximation to α, use the Newton-Raphson method to find a second approximation, x_2, to α. Give your answer to three decimal places.

[4 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE	Answer space for question 2

3 A curve C has equation $y = 2x(27 - x^2)$.

The point P on the curve C has coordinates $(-3, -108)$.

The point Q on the curve C has x-coordinate $-3 + h$.

(a) Find the gradient of the line PQ, giving your answer in its simplest form.

[3 marks]

(b) (i) Use your answer to part **(a)** to determine whether P is a stationary point on the curve C. Explain your reasoning.

[2 marks]

(ii) Write down an equation of the tangent to C at the point P.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE	Answer space for question 3

4 The quadratic equation

$$2x^2 + 3x + k = 0$$

where k is a real constant, has roots α and β. It is given that $\alpha^2 + \beta^2 = -\frac{7}{4}$.

(a) Explain why the statement $\alpha^2 + \beta^2 = -\frac{7}{4}$ implies that α and β cannot both be real.

[2 marks]

(b) Write down the value of $\alpha + \beta$.

[1 mark]

(c) Show that $k = 4$.

[2 marks]

(d) Find a quadratic equation, with integer coefficients, which has roots $\alpha^2 + \frac{1}{\beta}$ and $\beta^2 + \frac{1}{\alpha}$.

[5 marks]

QUESTION PART REFERENCE	**Answer space for question 4**

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QUESTION PART REFERENCE	Answer space for question 4

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5 It is given that $z = p + 3\mathrm{i}$ where p is a real number.

It is also given that $w = z^2 - 8z^* - 18p^2\mathrm{i}$.

(a) Find, in terms of p, the real part and the imaginary part of w.

[5 marks]

(b) Given that w is purely imaginary, show that there is only one possible non-zero value of w and state this value.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 5

QUESTION PART REFERENCE	Answer space for question 5

6 (a) Find the general solution of the equation

$$\cos(x - 38°) + \cos 80° = 0$$

giving your answer in degrees in a simplified form.

[4 marks]

(b) It is given that

$$\cos\frac{5\pi}{12} = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

(i) Find the exact value of $\cos^2\left(\frac{5\pi}{12}\right)$.

[1 mark]

(ii) Hence express $\cos^2\left(\frac{5\pi}{12}\right)$ in the form $\left(\sin\frac{\pi}{6}\right)\left(\sin(a\pi) + \sin(b\pi)\right)$, where a and b are **positive** rational numbers.

[3 marks]

QUESTION PART REFERENCE

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7 (a) Use the formulae for $\sum_{r=1}^{n} r^3$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n} r(4r+1)(4r-1) - 12n\sum_{r=1}^{n} r^2 = \frac{n}{2}(n+1)(4n-1)$$

[4 marks]

(b) Hence show that there is exactly one value of n for which

$$\sum_{r=1}^{n}(16r^3 - r - 57) - 12n\sum_{r=1}^{n} r^2 = 0$$

stating the value of n.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 7

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8 (a) The matrices $\mathbf{A}$ and $\mathbf{B}$ are defined by

$$\mathbf{A} = \begin{bmatrix} p & 5-2p \\ 25-\dfrac{3p}{2} & 15 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix}$$

Given that $\mathbf{A} + q\mathbf{B} = n\mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix, find the values of the constants p, q and n.

[5 marks]

(b) (i) Write down the 2×2 matrix which represents a stretch parallel to the x-axis of scale factor 3.

[1 mark]

(ii) Find the 2×2 matrix which represents a reflection in the line $\sqrt{3}x - y = 0$, using surd forms where appropriate.

[2 marks]

(iii) Hence find the matrix which represents the combined transformation of a stretch parallel to the x-axis of scale factor 3 followed by a reflection in the line $\sqrt{3}x - y = 0$.

[2 marks]

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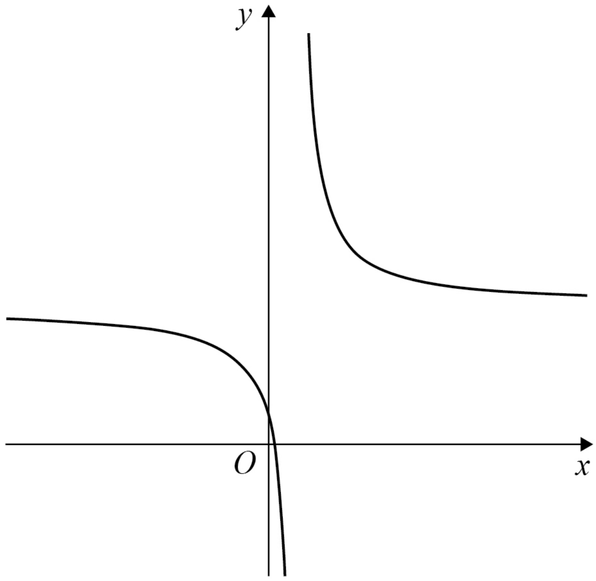
9 The diagram shows a sketch of a curve.



The equation of the curve is $y = \frac{5x - 1}{x - 1}$.

(a) Write down the coordinates of the point where the curve intersects the x-axis.

[1 mark]

(b) Given that the line $y = -x + c$ intersects the curve, show that the x-coordinates of the points of intersection must satisfy the equation

$$x^2 + (4 - c)x + c - 1 = 0$$

[3 marks]

(c) (i) **Hence** find the equations of the two tangents to the curve that are parallel to the line $y = -x$.

(No credit will be given for solutions based on differentiation.)

[4 marks]

(ii) The two tangents touch the curve at the points A and B. Show that AB and parts of the two tangents can form three sides of a square and find the area of the square.

[6 marks]

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